Up and Down Staircases

Extension activities

Idea 1

John Harrison (the inventor of the Numdrum - see his website) offers the following as an extension to the Up and Down Staircases problem:

If we write the number of blocks in each column of the staircase, we get a series of numbers which I call the Noble Duke of York numbers.

For instance in the case of a staircase of height 5 blocks, we get:
1 2 3 4 5 4 3 2 1

If we assume these digits have place value (i.e. the list above becomes the number 123454321 – one hundred and twenty three million, four hundred and fifty four thousand, three hundred and twenty one), then amazingly we find that the square root of this number is:
11111 (i.e. five 1s)

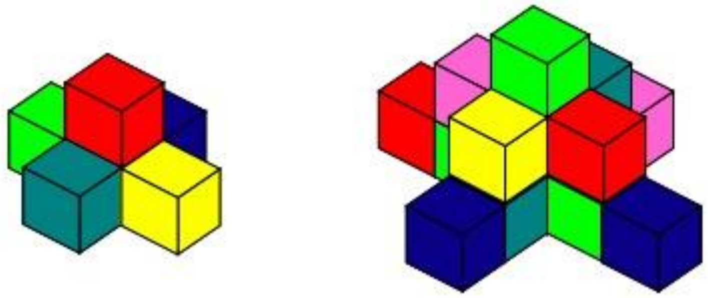
In fact the square roots of all the Noble Duke of York numbers are a string of 1s, the number of 1s in the string is equal to the centre (largest) digit of the staircase.

Children could be invited to investigate the square root of these numbers and to find a connection between the number of 1s in the square root and the number itself.

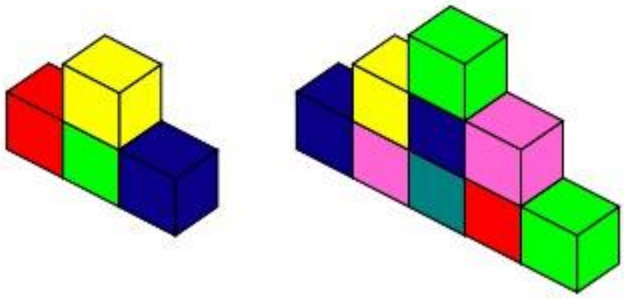
Idea 2

Bernard Bagnall suggests:

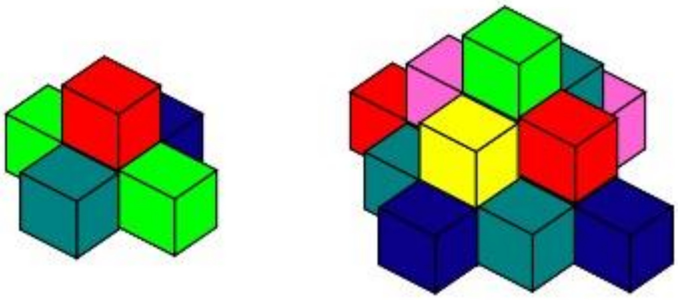
This challenge can also be extended by asking the question "I wonder what would happen if we change the stairs slightly?". Sometimes you have steps up to a good sight-seeing place (for example), four small sets of steps, each at right angles to the other. So we'd have a set of steps coming from North, South, East and West. The first two might look like:



whereas the first set looked like this:



Or a third different set could have "infill" - steps in between (health and safety!):



Then learners can explore 3 sets of numbers that show the number of cubes required for many of each set.

	First	diff	diff	DR	NSEW	diff	diff	DR	Infill	diff	diff	diff	DR
1	1			1	1			1	1				1
2	4	3	2	4	6	5	4	6	6	5	8		6
3	9	5	2	9	15	9	4	6	19	13	12	4	1
4	16	7	2	7	28	13	4	1	44	25	16	4	8
5	25	9	2	7	45	17	4	9	85	41	20	4	4
6	36	11	2	9	66	21	4	3	146	61	24	4	2
7	49	13	2	4	91	25	4	1	231	85	28	4	6
8	64	15	2	1	120	29	4	3	344	113	32	4	2
9	81	17	2	9	153	33	4	9	489	145	36	4	3
10	100	19	2	1	190	37	4	1	670	181	40	4	4
11	121	21	2	4	231	41	4	6	891	221	44	4	9
12	144	23	2	9	276	45	4	6	1156	265	48	4	4
13	169	25	2	7	325	49	4	1	1469	313	52	4	2
14	196	27	2	7	378	53	4	9	1834	365	56	4	7
15	225	29	2	9	435	57	4	3	2255	421	60	4	5
16	256	31	2	4	496	61	4	1	2736	481	64	4	9
17	289	33	2	1	561	65	4	3	3281	545	68	4	5
18	324	35	2	9	630	69	4	9	3894	613	72	4	6
19	361	37	2	1	703	73	4	1	4579	685	76	4	7
20	400	39		4	780	77		6	5340	761			3

The last column in each shows the digital root of the numbers in the first column of each. See the article [Digital Roots](). Lots of things to explore here!

Generally speaking once children have got two or three sets of results that they've found by slightly changing the rules (as above) and they've done some exploring, then it's a good idea to compare. In the results we have here they can look at the numbers required for FIRST and subtract those results from the other two sets of results, as well as subtracting the NSEW results from the INFILL results.

So, for example, the results would be:

INFILL minus NSEW

answer	DR
0	9
0	9
4	4
16	7
40	4
80	8
140	5
224	8
336	3
480	3
660	3
880	7
1144	1
1456	7
1820	2
2240	8
2720	2
3264	6
3876	6
4560	6
5320	1
6160	4
7084	1
8096	5
9200	2
10400	5
11700	9
13104	9
14616	9
16240	4

INFILL minus FIRST

answer	DR
0	9
2	2
10	1
28	1
60	6
110	2
182	2
280	1
408	3
570	3
770	5
1012	4
1300	4
1638	9
2030	5
2480	5
2992	4
3570	6
4218	6
4940	8
5740	7
6622	7
7590	3
8648	8
9800	8
11050	7
12402	9
13860	9
15428	2
17110	1

Then pupils can look at their digital roots.

I noticed a number of things but just taking an example, looking at the Digital Roots, start with the 2nd 9 in the first set and the 1st 2 in the next set going down three at a time we add on 4. [Note that in Digital Roots you have 7 + 4 = 2 and 9 + 4 = 3 etc.]

9	4	7	**4**	8	5	**8**	3	3	**3**	7	1	**7**	2
2	1	1	**6**	2	2	**1**	3	3	**5**	4	4	**9**	5

So learners now could have three number sequences to explore separately or together. Those pupils able to use spreadsheets could pursue thoughts in that way.